



Competition #9

The Junior Online Math Olympiad

Aditya Raut, Michael Tang, Zi Song Yeoh,
Guilherme Dela Corte, Happy Melodies Sim

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Short Questions

1. Without repetition, Guilherme uses the digits 1, 3, 5, 7 and 9 to make numbers greater than 3900. How many numbers can he make?
2. Let $2014^{2014} = x$ Given that

$$x \pmod{14} = J$$

$$x \pmod{17} = O$$

$$x \pmod{(14 + 17)} = M$$

and \overline{ABC} means the number in decimal with digits A,B and C. Then find the remainder when $\overline{JOMO}^{\overline{JOMO}}$ is divided by 2014.

3. Between the 2 numbers 1 and 2^{121} , 10 geometric means are inserted (G_1, G_2, \dots, G_{10}) Between the consecutive geometric means and the numbers, 10 geometric means are inserted. There are 11 such gaps, the one between 1 and G_1 , the second between G_1 and G_2 , so on till the 11th gap between G_{10} and 2^{121} .

In each of these 11 gaps, we insert 10 geometric means, making 110 new geometric means g_1, g_2, \dots, g_{110} .

If the numbers J, O, M are as found in the previous question, then find the value of

$$\log_2 G_J + \log_2 G_O + \log_2 G_M + \log_2 G_O + \log_2 g_{\overline{JO}} + \log_2 g_{\overline{MO}} + \overline{JOMO}$$

4. How many positive integers n are there so that the base-4 representation of n has 8 digits, and the product of the nonzero digits is 144?
5. Mark and Spencer both arrived at the airport anytime from 5pm to 7pm. Mark will only spend 40 minutes in the airport before leaving, while Spencer spends 15 minutes before leaving. The probability that they will meet is $\frac{a}{b}$ for coprime positive integers a and b . Find the value of $a + b$.
6. Let $P(x)$ be a polynomial with real coefficients such that for all real x , $P(x^2) + P(x) = P(P(x))$. Find the sum of all possible values of $P(1)$.
7. The smallest positive root to the equation $\lceil x \rceil - \lfloor x \rfloor = \tan x$ is R . Evaluate $\frac{\pi}{R}$.
8. Adi is climbing a staircase. But for safety purpose, Sam has allowed him to climb up only 1 or 2 or 3 stairs at a time. Then find the number of ways in which Adi can climb up the staircase of 9 stairs.
9. Find the remainder when $7201^{7200^{7199^{\dots^{2^1}}}}$ is divided by 2015.
10. ZS, Melody and Adi went for a birthday party at Yan Yau's house. After the party, they all depart to their respective houses. Adi walks home, with a constant speed a , while ZS and Melody rides a bike and drives home respectively with constant speeds b, c respectively. ZS's house is exactly halfway between Adi's house and Melody's house. They all depart simultaneously on 6pm. Melody arrive at her house at 6:30pm, only to remember that she had to get an important parcel from Cody whose house is exactly halfway between Adi's house and Yan Yau's house. Poor Melody, she departs immediately from her house, this time with a constant speed d . Melody arrived at Cody's house 15 minutes later. ZS arrived at her house at 7pm, while Anna finally arrives at her house at 7.30pm. Their houses all lie on a single street. When did Melody first meet ZS after the party? Suppose $a < b < c < d$.

Long Questions

Explain your answer for each question.

1. Show that the sum of the fifth powers of five consecutive integers is always divisible by 25.
2. Evaluate all roots of the equation $(x - 3)^4 + (x - 2)^4 = 1$.
3. Prove that for all positive integers n , $4n^3 + 6n^2 + 4n + 1$ is not a perfect fourth power.