



Weekly Challenge 2015

The Junior Online Math Olympiad

Challenges

Week 4: (ZS) Let a, b, c be positive real numbers such that $abc = 1$. Prove that

$$\sum \frac{a^2 + b^2}{a^8 + b^8} \leq a^3 + b^3 + c^3$$

(3 points)

Week 3: (Michael Tang) Prove that there is no pair of real numbers (a, b) so that both of the systems

$$\begin{cases} x^2 + y^2 = a \\ xy = b \end{cases}$$

$$\begin{cases} x^2 + y^2 = b \\ xy = a \end{cases}$$

have positive real solutions for (x, y) .

(3 Points)

Week 2: (Navi) Prove that for any positive integers $x \neq y$, there exist positive integers (m, n) such that:

$$\frac{x^4 + y^4 + m^4}{x^2 + y^2 + m^2} = m^2 + n$$

(2 Points)

Week 1: (ZS) Prove that $10a^2 + 9b^2$ and $10b^2 + 9a^2$ can't simultaneously be perfect squares for all positive integers a, b .

(3 points)